

Computation of Fractions: Considerations for Instruction

Purpose and Overview of Guide

The purpose of this guide is to provide strategies and materials for developing and implementing lessons for students who need intensive instruction in the area of fractions. Special education teachers, mathematics interventionists, and others working with students struggling in the area of fractions may find this guide helpful.

Within college- and career-ready standards, fractions are taught in Grades 3–5. This guide may be used as these concepts are introduced or with students at higher grade levels who continue to struggle with the concepts. Sample activities, worksheets, and supplemental materials also accompany this guide and are available for download at <http://www.intensiveintervention.org>.

The guide is divided into four sections:

1. The sequence of skills as defined by college- and career-ready standards
2. A list of important vocabulary and symbols
3. A brief explanation of the difficulties that students may have with fractions
4. Suggested strategies for teaching fraction computation concepts

Sequence of Skills—College- and Career-Ready Standards

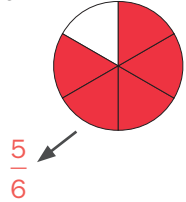
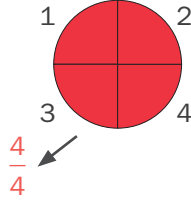
Build fractions from unit fractions—applying and extending operations of whole numbers.

- Add and subtract fractions.
- Decompose fractions in more than one way.
- Add and subtract mixed numbers with like denominators.
- Solve word problems involving addition or subtraction.
- Multiply fractions.
- Solve word problems involving multiplication.

- Use equivalent fractions.
- Add and subtract fractions with unlike denominators.
- Solve word problems involving addition or subtracting of unlike denominators.
- Divide fractions.
- Solve word problems involving division of fractions.
- Continue multiplication.

Vocabulary and Symbols

The following terms are important for students to understand when working with fractions:

| | | |
|--|--|--|
| <p>Fraction: A part of a whole, with all parts equivalent.</p> $\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{1}{8}, \frac{4}{5}$ | <p>Numerator: How many parts of the whole.</p>  | <p>Denominator: How many parts make up the whole.</p>  |
| <p>Common Denominator: One or more fractions have the same denominator. Necessary for adding and subtracting fractions.</p> $\frac{1}{8} + \frac{2}{8}$ | <p>Equivalent Fractions: Fractions with equal value.</p> $\frac{4}{6} = \frac{2}{3} \quad \frac{2}{8} = \frac{1}{4}$ | <p>Simplify/Reduce: Putting fractions in lowest terms.</p> $\frac{12}{15} = \frac{3 \times 4}{3 \times 5} = \frac{4}{5}$ |
| <p>Least Common Multiple (LCM): The smallest common multiple of two or more denominators. Used to determine common denominator.</p> <p>Multiples of 3: 3, 6, 9, 12, 15</p> <p>Multiples of 5: 5, 10, 15, 20</p> <p>LCM is 15.</p> | <p>Greatest Common Factor (GCF): Largest common factor for the numerator and the denominator. Used to simply/reduce fractions.</p> <p>Factors of 12: 1, 2, 3, 4, 6, 12</p> <p>Factors of 15: 1, 3, 5, 15</p> <p>GCF is 3.</p> | <p>Proper Fraction: A fraction that is less than one.</p> $\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{1}{8}, \frac{4}{5}$ |
| <p>Improper Fraction: A fraction that is greater than one.</p> $\frac{6}{4}, \frac{9}{6}, \frac{14}{5}, \frac{8}{2}$ | <p>Mixed Number: A number that has a whole number <i>and</i> a fraction.</p> $4 \frac{1}{4} \quad 10 \frac{2}{3}$ | <p>Unit Fraction: A fraction with 1 in the numerator.</p> $\frac{1}{12}, \frac{1}{8}, \frac{1}{5}, \frac{1}{3}, \frac{1}{2}$ |

Common Areas of Difficulty

Prerequisite Skills Not Mastered

- Basic fact retrieval (for computation and comparison of fractions with unlike denominators)

Specific Fraction Skills

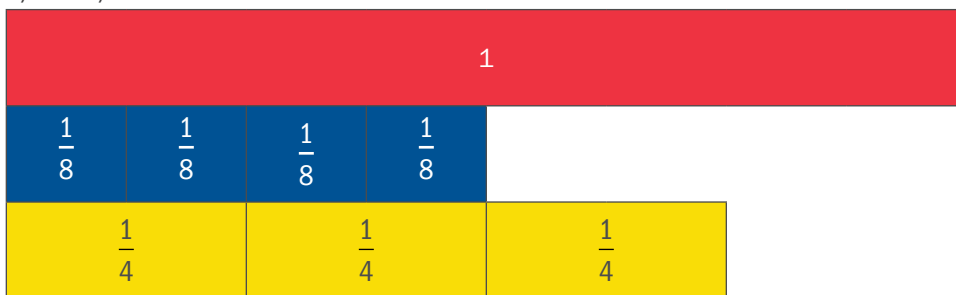
- Reading fractions
- Writing fractions
- Understanding that the larger the denominator, the smaller the value
- Poor understanding of multiples
- Understanding the four models of fractions and when to use them:
 - Area
 - Sets
 - Measurement
 - Division

Conceptual Understanding

Fraction tiles and **fraction circles** can be used to help students visualize and conceptually understand many fraction concepts. These manipulatives represent 1 whole, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{12}$.

Adding Fractions

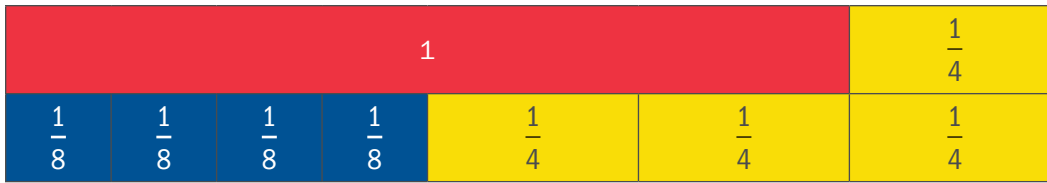
$$\frac{4}{8} + \frac{3}{4} =$$



Start with the whole.

Place $\frac{4}{8}$ under the whole.

Place $\frac{3}{4}$ under the $\frac{4}{8}$.



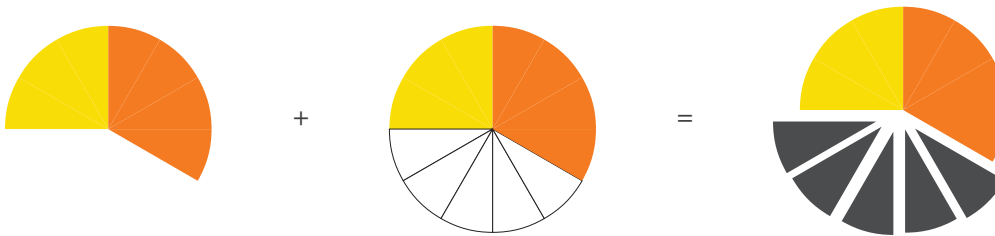
Combine $\frac{4}{8} + \frac{3}{4}$ under the whole.

Students should realize that $\frac{1}{4}$ more is needed to complete the problem.

$\frac{4}{8} + \frac{3}{4} = 1$ and $\frac{1}{4}$.

The same can be done using fraction circles.

$\frac{1}{4} + \frac{1}{3}$



Start with $\frac{1}{4}$ and $\frac{1}{3}$ pieces.

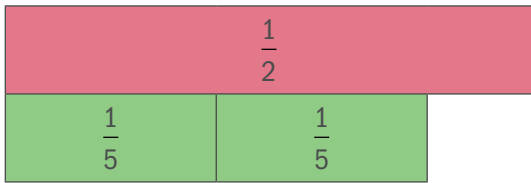
Place the pieces on the whole circle.

Determine that $\frac{1}{12}$ pieces need to be used.

Show that $\frac{5}{12}$ of the whole remains, which means $\frac{1}{4} + \frac{1}{3}$ equals $\frac{7}{12}$.

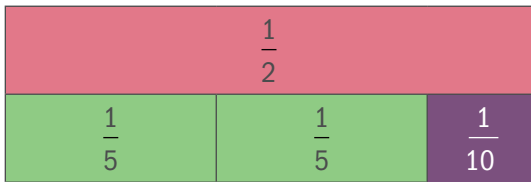
Subtracting Fractions

$$1/2 - 2/5 =$$



Start with $1/2$ and show subtracting $2/5$.

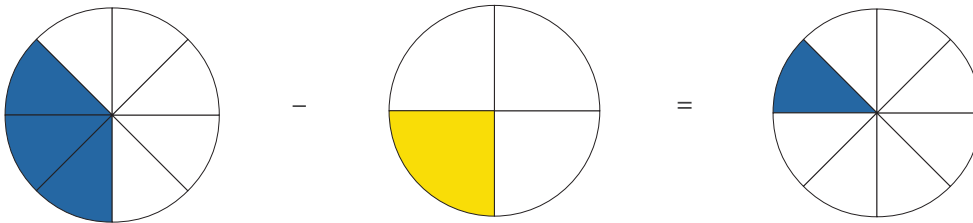
$1/2 - 2/5$ is the difference between a pink tile and two green tiles.



Place a $1/10$ next to $2/5$.

The difference is one purple tile, or $1/10$.

$$3/8 - 1/4$$



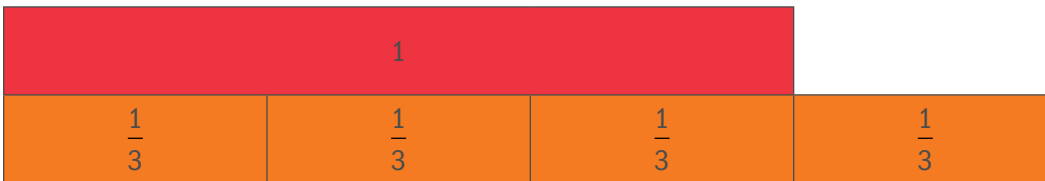
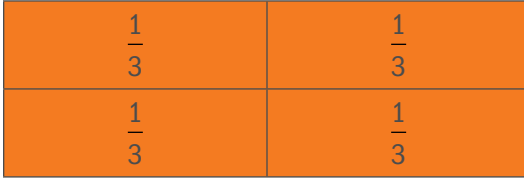
Start with $3/8$ and show subtracting $1/4$.

This shows that $1/4$ is equal to $2/8$, so the difference is $1/8$.

Multiplying Fractions

$$2 \times \frac{2}{3}$$

Show two sets of $\frac{2}{3}$ tiles.



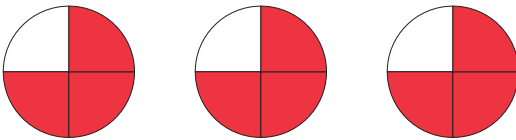
Show 1 whole.

Show two sets of $\frac{2}{3}$ tiles.

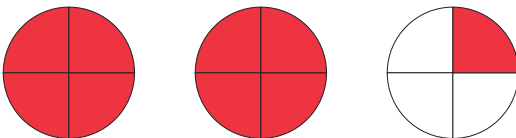
This shows $2 \times \frac{2}{3}$ equals 1 and $\frac{1}{3}$.

$$3 \times \frac{3}{4}$$

Show three sets of $\frac{3}{4}$ tiles.



Combine the sets, showing that they are equal to 2 and $\frac{1}{4}$.



Note to Teachers: Fraction tiles and circles will not work with all problems. You should ensure that the manipulatives will work for the problems you plan to demonstrate. For example, you cannot use these manipulatives to show $\frac{1}{5} + \frac{5}{6} = \frac{26}{30}$ because you do not have pieces that show 30ths.

Representing Multiplication of Fractions With Grids

$$2/3 \times 1/4$$

| | | | |
|--|--|--|--|
| | | | |
| | | | |
| | | | |

Using the denominators, build a grid—the first denominator represents the number of rows, and the second denominator represents the number of columns.

The numerator in each fraction tells how many rows and columns to shade in.

For $2/3$, shade in two rows.

For $1/4$, shade in one column.

The total number of boxes will be your new denominator: 12.

The total number of boxes shaded for both fractions will be your new numerator:

$$2/3 \times 1/4 = 2/12.$$

Using Fraction Bars to Determine Least Common Denominator

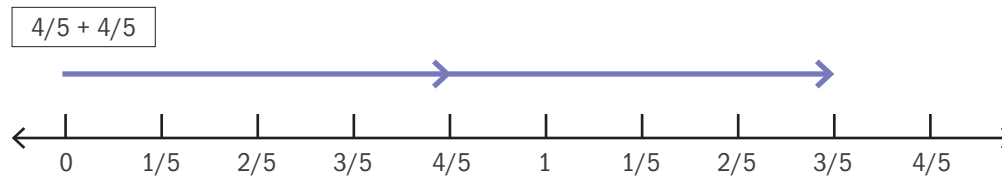
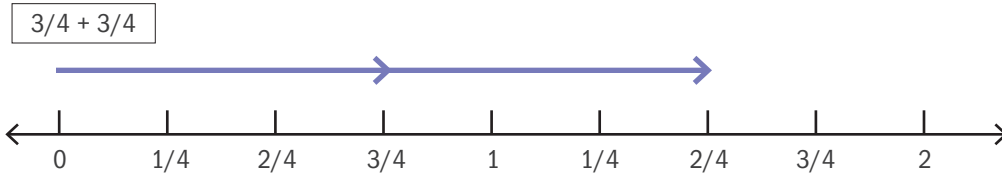
Each bar has a number, 1–9, at the beginning, and each of its multiples is listed (up to $\times 9$) on the bar.

$$2/3 + 3/4 =$$

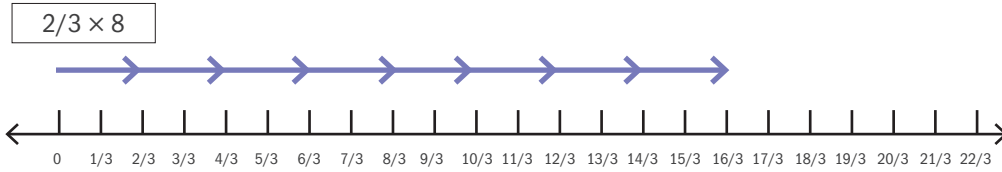
| | | | | | | | | | |
|----------|---|---|----|----|----|----|----|----|----|
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |

1. Select the fraction bars for 3 and 4. Line them up under each other.
2. By looking at the two fraction bars, determine the LCM, which is 12.
3. Convert each fraction to a new fraction with the denominator of 12.
 - a. $2/3 = ?/12 \rightarrow 12 \div 3 = 4 \rightarrow 4 \times 2 = 8 \rightarrow 2/3 = 8/12$
 - b. $3/4 = ?/12 \rightarrow 12 \div 4 = 3 \rightarrow 3 \times 3 = 9 \rightarrow 3/4 = 9/12$
4. $2/3 + 3/4$ now becomes $8/12 + 9/12$.
5. $8/12 + 9/12 = 17/12$
6. $17/12$ can be reduced to 1 and $5/12$.

Adding Fractions With a Number Line



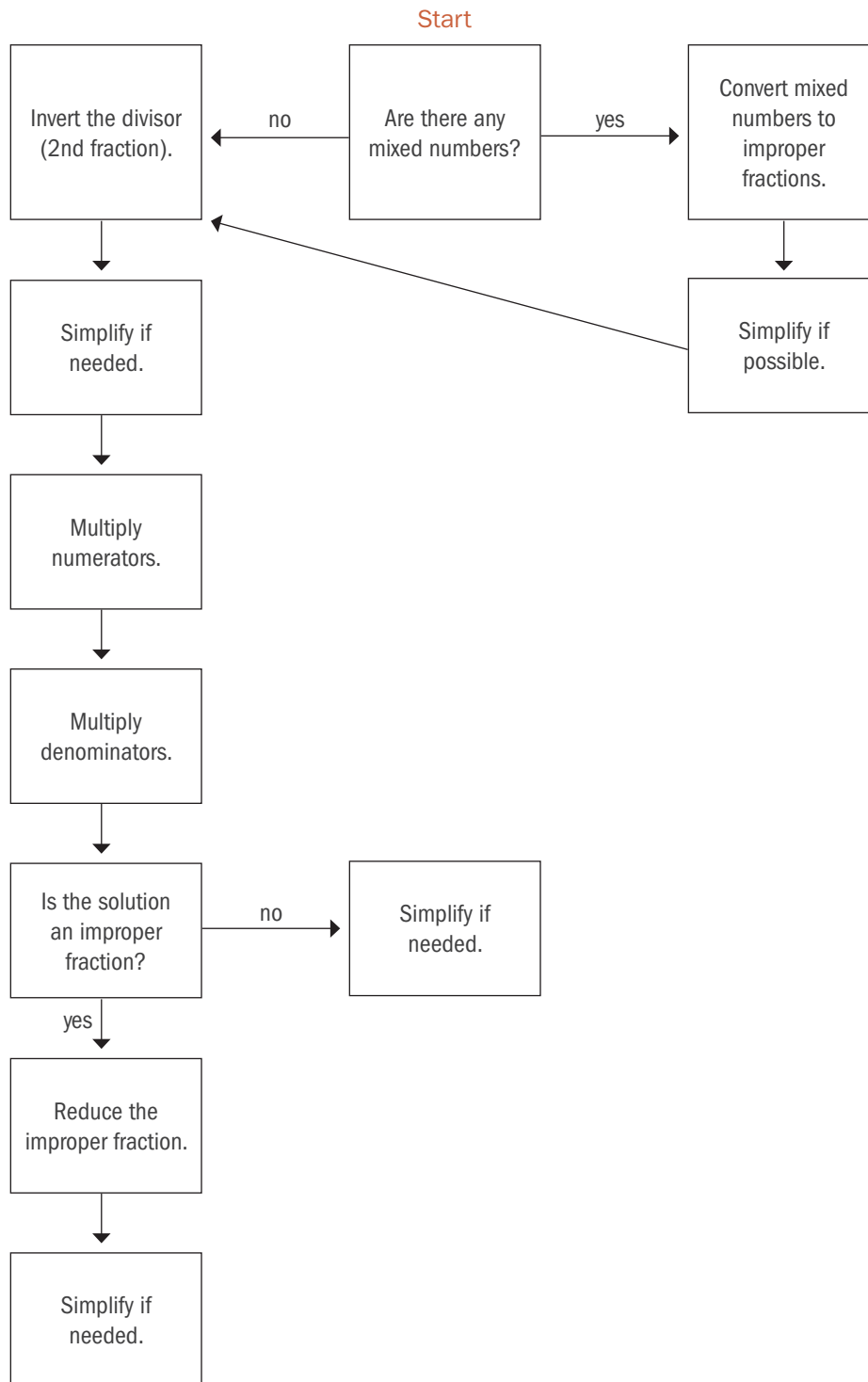
Multiplying Fractions With a Number Line



Procedural Understanding

Flowcharts can assist students in understanding the algorithm of a concept.

Flowchart for Dividing Fractions



Algorithms

Knowing **algorithms** is an important component for solving fraction problems. Students must understand the steps needed to solve each type of problem. Teachers may use these step-by-step procedures to determine where a student may be having difficulty in the algorithm. After a teacher has determined where the issue is, then teaching can begin at that point.

Adding Fractions With Like Denominators

- (a) Make sure the denominators are the same.
- (b) Add the numerators.
- (c) Put the answer over the same denominator.

(a) $\frac{3}{5} + \frac{1}{5}$

(b) $\frac{3 + 1}{5}$

(c) $\frac{4}{5}$

Subtracting Fractions With Like Denominators

- (a) Make sure the denominators are the same.
- (b) Subtract the second numerator from the first.
- (c) Put the answer over the same denominator.

(a) $\frac{7}{12} - \frac{5}{12}$

(b) $\frac{7 - 5}{12}$

(c) $\frac{2}{12}$

Adding Fractions With Unlike Denominators

- (a) Find the least common denominator (LCD) of the fractions: $4 \times 3 = 12$
- (b) Rename the first fraction so it has the LCD.
- (c) Rename the second fraction so it has the LCD.
- (d) Rewrite the problem with the new fractions.

- (e) Add the numerators.
- (f) Put the answer over the LCD.

$$\frac{1}{4} + \frac{2}{3}$$

(a) $4 \times 3 = 12 = \text{LCD}$

(b) $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$

(c) $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

(d) $\frac{3}{12} + \frac{8}{12}$

(e) $\frac{3 + 8}{12} =$

(f) $\frac{11}{12}$

Subtracting Fractions With Unlike Denominators

- (a) Find the LCD of the fractions.
- (b) Rename the first fraction so it has the LCD.
- (c) Rename the second fraction so it has the LCD.
- (d) Rewrite the problem with the new fractions.
- (e) Subtract the second numerator from the first.
- (f) Put the answer over the LCD.

$$\frac{3}{5} - \frac{1}{2} =$$

(a) $5 \times 2 = 10 = \text{LCD}$

(b) $\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$

(c) $\frac{1 \times 5}{2 \times 5} = \frac{5}{10}$

(d) $\frac{6}{10} - \frac{5}{10}$

(e) $\frac{6 - 5}{10} =$

(f) $\frac{1}{10}$

Multiplying Fractions

- (a) Multiply the numerators of the fractions.
- (b) Multiply the denominators of the fractions.
- (c) Put the product of the numerators on top of the fraction.
- (d) Put the product of the denominators on the bottom of the fraction.

$$\frac{3}{8} \times \frac{3}{4} =$$

(a) $3 \times 3 = 9$

(b) $8 \times 4 = 32$

(c, d) $\frac{9}{32}$

Dividing Fractions

- (a) Invert the second fraction (the divisor).
- (b) Change the division sign to a multiplication sign.
- (c) Multiply the numerators of the fractions.
- (d) Multiply the denominators of the fractions.
- (e) Put the product of the numerators on top of the fraction.
- (f) Put the product of the denominators on the bottom of the fraction.

(a) $\frac{2}{5} \div \frac{3}{4}$

(b) $\frac{2}{5} \times \frac{4}{3}$

(c, d) $\frac{2 \times 4}{5 \times 3} =$

(e, f) $\frac{8}{15}$

Adding Mixed Numbers

- (a) Convert the first mixed number to an improper fraction.
- (b) Multiply the denominator by the whole number.
- (c) Put the product over the denominator.
- (d) Add the new fraction's numerator to the original fraction's numerator.
- (e) Use the same denominator for the new numerator.
- (f) Convert the second mixed number to an improper fraction.
- (g) Multiply the denominator by the whole number.
- (h) Put the product over the denominator.
- (i) Add the new fraction's numerator to the original fraction's numerator.
- (j) Use the same denominator for the new numerator.
- (k) Rewrite the problem with improper fractions.
- (l) Find the LCD of the fractions.
- (m) Rename the first fraction so it has the LCD.
- (n) Rename the second fraction so it has the LCD.
- (o) Rewrite the problem with the LCD.
- (p) Add the numerators.
- (q) Put the answer over the LCD.
- (r) Simplify. Reduce improper fraction to a mixed number.

$$2\frac{3}{4} + 6\frac{3}{8}$$

$$(a) 2\frac{3}{4}$$

$$(b) 4 \times 2 = 8$$

$$(c) \frac{8}{8}$$

$$(d) \frac{4}{8} + 3 =$$

$$(e) \frac{11}{4}$$

$$(f) 6\frac{3}{8}$$

$$(g) 8 \times 6$$

$$(h) = \frac{48}{8}$$

$$(i) \frac{48 + 3}{8} =$$

$$(j) \frac{51}{8}$$

$$(k) \frac{11}{4} + \frac{51}{8}$$

$$(l) 4 \times 2 = 8, 8 \times 1 = 8 = \text{LCD}$$

$$(m) \frac{11 \times 2}{4 \times 2} = \frac{22}{8}$$

$$(n) \frac{51 \times 1}{8 \times 1} = \frac{51}{8}$$

$$(o) \frac{22}{8} + \frac{51}{8}$$

$$(p) \frac{22 + 51}{8} =$$

$$(q) \frac{73}{8} =$$

$$(r) 9\frac{1}{8}$$

Multiplying Mixed Numbers

- (a) Convert the first mixed number to an improper fraction.
- (b) Multiply the denominator by the whole number.
- (c) Put the product over the denominator.
- (d) Add the new fraction's numerator to the original fraction's numerator.
- (e) Use the same denominator for the new numerator.
- (f) Convert the second mixed number to an improper fraction.
- (g) Multiply the denominator by the whole number.
- (h) Put the product over the denominator.
- (i) Add the new fraction's numerator to the original fraction's numerator.
- (j) Use the same denominator for the new numerator.
- (k) Rewrite the problem with the improper fractions.
- (l) Multiply the numerators of the improper fractions.
- (m) Multiply the denominators of the improper fractions.
- (n) Put the product of the numerators on top of the fraction.
- (o) Put the product of the denominators on the bottom of the fraction.
- (p) Simplify. Reduce improper fraction to a mixed number.

Example:

$$3\frac{2}{9} \times 4\frac{3}{4} =$$

(a) $3\frac{2}{9}$

(b) $9 \times 3 = 27$

(c) $\frac{27}{9}$

(d) $\frac{27 + 2}{9} =$

(e) $\frac{29}{9}$

(f) $4\frac{3}{4}$

(g) $4 \times 4 = 16$

(h) $\frac{16}{4}$

(i) $\frac{16 + 3}{4} =$

(j) $\frac{19}{4}$

(k) $\frac{29}{9} \times \frac{19}{4}$

(l) $29 \times 19 = 551$

(m) $9 \times 4 = 36$

(n, o) $\frac{551}{36} =$

(p) $15\frac{11}{36}$

Dividing Mixed Numbers

- (a) Convert the first mixed number to an improper fraction.
- (b) Multiply the denominator by the whole number.
- (c) Put the product over the denominator.
- (d) Add the new fraction's numerator to the original fraction's numerator.
- (e) Use the same denominator for the new numerator.
- (f) Convert the second mixed number to an improper fraction.
- (g) Multiply the denominator by the whole number.
- (h) Put the product over the denominator.
- (i) Add the new fraction's numerator to the original fraction's numerator.
- (j) Use the same denominator for the new numerator.
- (k) Rewrite the problem with the improper fractions.
- (l) Invert the second fraction (the divisor).
- (m) Change the division sign to a multiplication sign.
- (n) Multiply the numerators of the fractions.
- (o) Multiply the denominators of the fractions.
- (p) Put the product of the numerators on the top of the fraction.
- (q) Put the product of the denominators on the bottom of the fraction.
- (r) Simplify. Reduce improper fraction to a mixed number.

Example:

$$6\frac{1}{2} \div 3\frac{2}{3} =$$

(a) $6\frac{1}{2}$

(b) $2 \times 6 = 12$

(c) $\frac{12}{2}$

(d) $\frac{12 + 1}{2} =$

(e) $\frac{13}{2}$

(f) $3\frac{2}{3}$

(g) $3 \times 3 = 9$

(h) $\frac{9}{3}$

(i) $\frac{9 + 2}{3} =$

(j) $\frac{11}{3}$

(k) $\frac{13}{2} \div \frac{11}{3}$

(l, m) $\frac{13}{2} \times \frac{3}{11}$

(n) $13 \times 3 = 39$

(o) $2 \times 11 = 22$

(p, q) $\frac{39}{22} =$

(r) $1\frac{17}{22}$