National Center on INTENSIVE INTERVENTION

at American Institutes for Research

# **Basic Facts**





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While permission to reprint this publication is not necessary, the citation should be:

National Center on Intensive Intervention. (2015). *Basic facts.* Washington, DC: U.S. Department of Education, Office of Special Education Programs, National Center on Intensive Intervention.

This document was produced under the U.S. Department of Education, Office of Special Education Programs, Award No. H326Q110005. Celia Rosenquist serves as the project officer. The views expressed herein do not necessarily represent the positions or polices of the U.S. Department of Education. No official endorsement by the U.S. Department of Education of any product, commodity, service or enterprise mentioned in this website is intended or should be inferred. at American Institutes for Research

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Mathematics Training Materials: Basic Facts Contents

### **Teaching Basic Facts:** Considerations for Instruction

### Purpose and Overview of Guide

The purpose of this guide is to provide strategies and materials for developing and implementing lessons for students who need intensive instruction in the area of **basic facts.** Special educators, mathematics interventionists, and others working with students struggling in the area of basic facts find this guide helpful.

Within college- and career-ready standards, Basic Facts are typically taught in grades K-4. This guide may be used as these concepts are introduced or with students in higher grade levels who continue to struggle with the concepts. Sample activities, worksheets, and supplemental materials also accompany this guide and are available for download at http://www.intensiveintervention.org.

The guide is divided into four sections:

- 1. Sequence of skills as defined by college- and career-ready standards.
- 2. A list of important vocabulary and symbols.
- 3. A brief explanation of the difficulties students may have with basic facts.
- 4. Suggested strategies for teaching basic facts.

### Sequence of Skills – College- and Career-Ready Standards

(Numbers in parentheses represents the grade level of the standard.)

#### Addition and subtraction concepts:

- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. (K)
- Solve addition and subtraction word problems within 10. (K)
- Decompose numbers less than or equal to 10 into pairs in more than one way. (K)
- For any number from 1 to 9, find the number that makes 10 when added to the given number. (K)
- Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. (1)





 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. (2)

#### Add and subtract:

- Fluently add and subtract within 5. (K)
- Apply properties of operations as strategies to add and subtract. (1)
- Understand subtraction as an unknown-addend problem. (1)
- Relate counting to addition and subtraction. (1)
- Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. (1)
- Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. (1)
- Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. (1)
- Fluently add and subtract within 20 using mental strategies. (2)
- By end of Grade 2, know from memory all sums of two one-digit numbers. (2)

#### Multiplication and division concepts:

- Determine whether a group of objects (up to 20) has an odd or even number of members; write an equation to express an even number as a sum of two equal addends. (2)
- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (2)
- Interpret products of whole numbers. (3)
- Interpret whole-number quotients of whole numbers. (3)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. (3)
- Interpret a multiplication equation as a comparison. (4)
- Represent verbal statements of multiplicative comparisons as multiplication equations. (4)

#### Multiply and divide:

 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. (3)

- Apply properties of operations as strategies to multiply and divide. (3)
- Understand division as an unknown-factor problem. (3)
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division. (3)
- By the end of Grade 3, know from memory all products of two one-digit numbers. (3)
- Multiply or divide to solve word problems involving multiplicative comparison. (4)

### Vocabulary and Symbols

The following terms are important for students to understand when working with basic facts.

Add: To combine amounts.	Addend: An amount combined to another amount in an addition problem. addend + addend = sum	<b>Sum:</b> The total in an addition problem. addend + addend = sum
Plus sign: +	<b>Subtract:</b> To find the difference between two amounts; to take away.	<b>Minuend:</b> The starting amount in a subtraction problem. minuend – subtrahend = difference
<b>Subtrahend:</b> The take away amount in a subtraction problem. minuend – subtrahend = difference	<b>Difference:</b> The end result in a subtraction problem. minuend – subtrahend = difference	Minus sign: —
<b>Multiply:</b> To increase an amount a number of times.	<b>Factor:</b> An amount multiplied by another amount. factor × factor = product	<b>Product:</b> The end result in a multiplication problem. factor × factor = product
Multiplication sign: X Sometimes, * is used.	<b>Divide:</b> To break an amount into equal groups.	<b>Dividend:</b> The starting amount in a division problem. dividend ÷ divisor = quotient
<b>Division:</b> The number of groups in a division problem. dividend ÷ divisor = quotient	<b>Quotient:</b> The end result in a division problem. dividend ÷ divisor = quotient	Division sign: ÷ Sometimes, / or a fraction bar is used.
Additive Problem types relating to addition and subtraction.	<b>Multiplicative:</b> Problem types relating to multiplication and division.	Equal sign: = Should be interpreted as "the same as."

### **Basic Facts: Considerations for Instruction**

### **Common Areas of Difficulty**

#### Prerequisite skills not mastered:

- Knowledge of numbers and what number represent.
- Counting.

#### Specific Basic Facts skills:

- Understanding symbols.
- Fluency with facts.
- Strategies to calculate answers.

### About the Basic Facts

- Addition: 100 facts. Single-digit addend + single-digit addend = sum.
  - 3 + 4 = 7; 9 + 8 = 17
- Subtraction: 100 facts. Minuend single-digit subtrahend = single-digit difference.
  - 8-2=6; 13-9=4
- Multiplication: 100 facts. Single-digit factor × single-digit factor = product.
  - 2 × 7 = 14; 8 × 6 = 48
- Division: 90 facts. Dividend ÷ single-digit divisor = single-digit quotient.
  - 9 ÷ 3 = 3; 56 ÷ 7 = 8

### **Developing Conceptual Understanding**

**Manipulatives** can be used to help students understand the concepts behind the basic facts. Examples of manipulatives include: Unifix cubes, plastic clips, chips, or dominoes.



### Activities and Strategies Related to Specific Standards

#### **Represent addition and subtraction. (K)**

- Use objects (e.g., chips, fingers) to show two groups. Put the two groups together.
- Use objects (e.g., chips, fingers) to show one group. Take an amount away from the group.
- Tell stories to show addition (i.e., putting together, adding on).
- Tell stories to show subtraction (i.e., taking away, comparing).
- Use a number line. Move forward for addition. Move backwards for subtraction.
   (Also, compare the difference between two numbers for subtraction.)

#### Decompose numbers less than or equal to 10 into pairs in more than one way. (K)

 Use two different colors of the same manipulative to show all possible combinations of a specific number.



Demonstrate commutative property of addition; the order of the addends does not matter. For example, 3 + 1 is the same as 1 + 3.

## For any number from 1 to 9, find the number that makes 10 when added to the given number. (K)

• Use a Tens Frame and manipulatives to teach all possible combinations that make 10.



- Use a set of 10 clips or 10 cubes to teach all combinations that make 10.
- Use fingers. Start with some (e.g., 3) fingers held up. Count up to 10.
- Learn operation symbols (plus, minus, and equal signs) and how to use equation notation.

#### Solve addition and subtraction word problems within 10. (K) Use addition and subtraction within 20 to solve word problems. (1) Use addition and subtraction within 100 to solve one- and two-step word problems involving situations. (1)

- Present word problems written and orally. Provide situations that involve:
  - Adding to

"Maisie has 4 buttons. She buys 3 buttons at the store. How many buttons does Maisie have now?"

Taking from

"Maisie had 9 buttons, and then her brother took 2 of them. How many buttons does Maisie have left?"

- Putting together
   "Maisie has 5 buttons. Jamey has 4 buttons. How many buttons do they have altogether?"
- Comparing "Maisie has 8 buttons and Jamey has 2 buttons. How many fewer buttons does Jamey have than Maisie?"
- Use stories where the unknown is in all positions.
  - For the comparing problem from above:
    - "Maisie has 8 buttons and Jamey has 2 buttons. How many fewer buttons does Jamey have than Maisie?" (unknown: difference)
    - "Maisie has 8 buttons. She has 6 more buttons than Jamey. How many buttons does Jamey have?" (unknown: subtrahend)

- "Jamey has 2 buttons. Maisie has 6 more buttons than Jamey. How many buttons does Maisie have?" (unknown: minuend)
- Act problems out with objects or students.

## Determine whether a group of objects (up to 20) has an odd or even number of members. (2)

- Practice skip counting by twos.
  - Start at 0 to skip count even numbers.
  - Start at 1 to skip count odd numbers.

#### Write an equation to express an even number as a sum of two equal addends. (2)

- Teach the addition doubles with rhymes or chants.
- Look at doubles patterns on an addition chart.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

## Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. (2)

Write an equation to express the total as a sum of equal addends. (2)

 Arrange objects in arrays and skip count rows or columns. This helps with learning multiplication as repeated addition.



Learn operation symbols (multiplication, division, and equal signs) and how to use equation notation.

#### Interpret products of whole numbers. (3)

 Use objects (e.g., chips and plates) to show a number of groups and the number within each group.



#### Interpret whole-number quotients of whole numbers. (3)

• Use objects (e.g., chips and plates) to show an amount divided evenly into groups.



Use multiplication and division within 100 to solve word problems. (3) Multiply or divide to solve word problems. (4)

- Present word problems written and orally. Provide situations that involve the following:
  - Equal groups
     "Lincoln has 5 buckets with 4 toy cars in each bucket. How many toy cars does Lincoln have?"
  - Comparison

"Lincoln has 4 toy cars. Roscoe has 7 times more cars than Lincoln. How many toy cars does Roscoe have?"

Combinations

"Lincoln has 4 shirts and 2 pairs of pants. What are all the combinations of shirts and pants?"

- Use stories where the unknown is in all positions of the problem.
  - For the equal groups problem from above: *"Lincoln has 5 buckets with 4 toy cars in each bucket. How many toy cars does*

Lincoln have?" (unknown: product;  $5 \times 4 = ?$ )

"Lincoln has 20 toy cars. He places the same number of cars in 5 buckets. How many cars are in each bucket?" (unknown: factor—number in each group;  $5 \times ? = 20$ )

"Lincoln has 20 toy cars. He wants to put 4 toy cars into each bucket. How many buckets does Lincoln need?" (unknown: factor—number of groups;  $? \times 4 = 20$ )

Act problems out with objects or students.

#### Fluently add and subtract within 5. (K)

### Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. (1)

Fluently add and subtract within 20 using mental strategies. (2) By end of Grade 2, know from memory all sums of two one-digit numbers. (2)

- Teach strategies for solving addition problems. Say, "Put the larger number in your head. Count on the smaller number by holding up a finger for each count. The sum is the last number you say."
- Teach strategies for solving subtraction problems. Say, "Put the subtrahend in your heard. Count up to the minuend. The difference is the number of counts."
- If a student answers incorrectly, encourage them to count to calculate the answer.
- Use number lines or addition tables if necessary.
- Practice with flashcards. Provide immediate feedback to incorrect facts.
- Practice within fact families. (3 + 4, 4 + 3, 7 3, 7 4)
- Practice with timed and untimed paper activities. Provide immediate feedback to incorrect facts.

#### Apply properties of operations as strategies to add and subtract. (1)

Teach the commutative property of addition. Teach how the commutative property can help students solve addition and subtraction facts. Say, "If you know 4 + 5 = 9, you also know that 5 + 4 = 9."

### Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. (1)

Show students a balance scale. Place objects on each side of the balance to represent addition and subtraction equations. Emphasize that both side of the balance must be the same.

#### Understand subtraction as an unknown-addend problem. (1) Relate counting to addition and subtraction. (1)

- Teach the reciprocal property of addition and subtraction. Say, "If you know 4 + 5 = 9, you know 9 4 = 5 and 9 5 = 4."
- Teach of thinking of subtraction as, "What number do I add to get to \_\_?" "Let's see.
   14 9 = \_\_. What number can I add to 9 to get 14? I start with 9 and count up.
   10, 11, 12, 13, 14. I added 5. 14 minus 9 is 5."

## Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. (1)

Introduce unknowns within stories and using manipulatives. For example, with a cup and chips: "I have 5 chips. There are some chips in the cup. If I have 8 chips altogether, how many chips are in the cup?"

## Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division. (3)

By the end of Grade 3, know from memory all products of two one-digit numbers. (3)

- Teach strategies for solving multiplication problems as repeated addition. Say, "7 times 3. That's 7 counted 3 times. 1, 2, 3, 4, 5, 6, 7. 8, 9, 10, 11, 12, 13, 14. 15, 16, 17, 18, 19, 20, 21. 7 times 3 is 21."
- If a student answers incorrectly, encourage them to count to calculate the answer.
- Use number lines or multiplication tables.
- Practice with flashcards. Provide immediate feedback to incorrect facts.
- Practice within fact families.  $(6 \times 4, 4 \times 6, 24 \div 6, 24 \div 4)$
- Practice with timed and untimed paper activities. Provide immediate feedback to incorrect facts.

## Determine the unknown whole number in a multiplication or division equation relating three whole numbers. (3)

Introduce unknowns within stories and using manipulatives.

#### Apply properties of operations as strategies to multiply and divide. (3)

• Teach the commutative property of multiplication. Teach how the commutative property can help students solve multiplication and division facts. Say, "If you know  $3 \times 8 = 24$ , you also know that  $8 \times 3 = 24$ ."

#### Understand division as an unknown-factor problem. (3)

- Teach the reciprocal property of multiplication and division. Say, "If you know  $7 \times 5 = 35$ , you know that  $35 \div 7 = 5$  and  $35 \div 5 = 7$ ."
- Teach of thinking of division as, "What number do I multiply to get to \_\_?" "Let's see.
   21 divided by 7. What number can I multiply to 7 to get to 21? I can count by 7. 7, 14, 21. I counted 7 3 times. 21 divided by 7 is 3."

## 2. Basic Facts: Addition and Subtraction Concepts

#### **Sample Activities**

- a) Activity One: Representing Addition and Subtraction with Objects
- b) Activity Two: Solve Subtraction Problems as an Unknown-Addend Problem

#### Worksheets

- a) Basic Facts: Addition Concepts
- b) Basic Facts: Subtraction Concepts





## **Sample Basic Facts:**

### Addition and Subtraction Concepts Activities 1–2

#### **College- and Career-Ready Standards Addressed:**

K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Understand and apply the relationship between addition and subtraction. 1.0A.4. Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.

### Activity One: Representing Addition and Subtraction with Objects

Purpose:	To add and subtract with concrete objects
Principles of Intensive Intervention	<ul> <li>Provide concrete learning opportunities (including use of manipulatives).</li> </ul>
Illustrated:	Provide explicit error correction, and have students repeat the correct process.
	Use precise, simple language to teach key concepts or procedures.
	<ul> <li>Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.</li> </ul>
	Provide repeated opportunities to practice each step correctly.
Materials:	<ul> <li>One-sided flashcards with addition and subtraction problems (addends should fall between 0 – 5; minuends should not exceed 10 and subtrahends should fall between 0 – 5; see Supplemental Materials)</li> </ul>
	<ul> <li>Concrete objects (teachers choice)</li> </ul>
	<ul> <li>Worksheet: Basic Facts: Addition Concepts (for extra practice)</li> </ul>
	<ul> <li>Worksheet: Basic Facts: Subtraction Concepts (for extra practice)</li> </ul>







Modeling Addition:	1.	Present addition problem $(2 + 5 = )$
	2.	Use colored bears or other concrete manipulative.
	3.	Show the number 2 by counting out 2 bears of the same color.
	4.	Place the two bears underneath the number 2.
	5.	Show the 5 by counting out 5 bears of a different color than the 2.
	6.	Place the five bears underneath the number 5.
	7.	Explain to students that the plus sign tells us to add.
	8.	Explain that when we add, we put things together.
	9.	Say, "To solve the problem, we put the 2 bears together with the 5 bears. Our answer is the number of bears. Let's count together. 1, 2, 3, 4, 5, 6, 7.
	10	Say, "The answer to $2 + 5 = 7$ ."
	11	. When we add, the amount gets bigger.
Guided Practice	1.	Teacher presents addition problem $(4 + 1 = )$
Guided Practice Addition:	1. 2.	Teacher presents addition problem $(4 + 1 = )$ Have students use colored bears or other concrete manipulative.
Guided Practice Addition:	1. 2. 3.	Teacher presents addition problem $(4 + 1 = )$ Have students use colored bears or other concrete manipulative. Students show the 4 by counting out 4 bears.
Guided Practice Addition:	1. 2. 3. 4.	Teacher presents addition problem (4 + 1 = ) Have students use colored bears or other concrete manipulative. Students show the 4 by counting out 4 bears. Students place the 4 bears underneath the "4."
Guided Practice Addition:	1. 2. 3. 4. 5.	<ul><li>Teacher presents addition problem (4 + 1 = )</li><li>Have students use colored bears or other concrete manipulative.</li><li>Students show the 4 by counting out 4 bears.</li><li>Students place the 4 bears underneath the "4."</li><li>Students show the number 1 by counting out 1 bear of a different color than the 4.</li></ul>
Guided Practice Addition:	1. 2. 3. 4. 5.	<ul> <li>Teacher presents addition problem (4 + 1 = )</li> <li>Have students use colored bears or other concrete manipulative.</li> <li>Students show the 4 by counting out 4 bears.</li> <li>Students place the 4 bears underneath the "4."</li> <li>Students show the number 1 by counting out 1 bear of a different color than the 4.</li> <li>Students place 1 bear underneath the number 1.</li> </ul>
Guided Practice Addition:	1. 2. 3. 4. 5. 6. 7.	<ul> <li>Teacher presents addition problem (4 + 1 = )</li> <li>Have students use colored bears or other concrete manipulative.</li> <li>Students show the 4 by counting out 4 bears.</li> <li>Students place the 4 bears underneath the "4."</li> <li>Students show the number 1 by counting out 1 bear of a different color than the 4.</li> <li>Students place 1 bear underneath the number 1.</li> <li>Students explain the plus sign tells us to add.</li> </ul>
Guided Practice Addition:	1. 2. 3. 4. 5. 6. 7. 8.	<ul> <li>Teacher presents addition problem (4 + 1 = )</li> <li>Have students use colored bears or other concrete manipulative.</li> <li>Students show the 4 by counting out 4 bears.</li> <li>Students place the 4 bears underneath the "4."</li> <li>Students show the number 1 by counting out 1 bear of a different color than the 4.</li> <li>Students place 1 bear underneath the number 1.</li> <li>Students explain the plus sign tells us to add.</li> <li>When we add, we put things together.</li> </ul>
Guided Practice Addition:	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> <li>5.</li> <li>6.</li> <li>7.</li> <li>8.</li> <li>9.</li> </ol>	<ul> <li>Teacher presents addition problem (4 + 1 = )</li> <li>Have students use colored bears or other concrete manipulative.</li> <li>Students show the 4 by counting out 4 bears.</li> <li>Students place the 4 bears underneath the "4."</li> <li>Students show the number 1 by counting out 1 bear of a different color than the 4.</li> <li>Students place 1 bear underneath the number 1.</li> <li>Students explain the plus sign tells us to add.</li> <li>When we add, we put things together.</li> <li>Students put the 4 bears together with the 1 bear.</li> </ul>
Guided Practice Addition:	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> <li>5.</li> <li>6.</li> <li>7.</li> <li>8.</li> <li>9.</li> <li>10</li> </ol>	<ul> <li>Teacher presents addition problem (4 + 1 = )</li> <li>Have students use colored bears or other concrete manipulative.</li> <li>Students show the 4 by counting out 4 bears.</li> <li>Students place the 4 bears underneath the "4."</li> <li>Students show the number 1 by counting out 1 bear of a different color than the 4.</li> <li>Students place 1 bear underneath the number 1.</li> <li>Students explain the plus sign tells us to add.</li> <li>When we add, we put things together.</li> <li>Students put the 4 bears together with the 1 bear.</li> <li>Students count the total.</li> </ul>
Guided Practice Addition:	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> <li>5.</li> <li>6.</li> <li>7.</li> <li>8.</li> <li>9.</li> <li>10</li> <li>11</li> </ol>	<ul> <li>Teacher presents addition problem (4 + 1 = )</li> <li>Have students use colored bears or other concrete manipulative.</li> <li>Students show the 4 by counting out 4 bears.</li> <li>Students place the 4 bears underneath the "4."</li> <li>Students show the number 1 by counting out 1 bear of a different color than the 4.</li> <li>Students place 1 bear underneath the number 1.</li> <li>Students explain the plus sign tells us to add.</li> <li>When we add, we put things together.</li> <li>Students put the 4 bears together with the 1 bear.</li> <li>Students count the total.</li> <li>Students say that the answer to 4 + 1 = 5.</li> </ul>

(Teacher should prompt students through the sequence of activities as needed.)

Modeling Subtraction:	1.	Present subtraction problem $(8 - 5 = )$
	2.	Use colored bears or other concrete manipulative.
	3.	Show the 8 by counting out 8 bears of the same color.
	4.	Explain to students that the minus sign tells us to subtract.
	5.	Explain that when we subtract, we take away.
	6.	To solve the problem, we take away 5 of the 8 bears. (Make sure the 5 bears are not a new set; students should be focused only on the 3 bears. This is important so the distinction between addition and subtraction is obvious.)
	7.	Say, "Let's count 5 of the bears to take away."
	8.	Teacher and students count 1, 2, 3, 4, 5.
	9.	Say, "Our answer is the number of bears we have left. Let's count together. 1, 2, 3."
	10	. Say, "The answer to $8 - 5 = 3$ ."
	11	. When we subtract, the amount gets smaller.
Guided Practice	1.	Teacher presents subtraction problem (6 - $2 = $ )
Subtraction:	2.	Student uses colored bears or other concrete manipulative.
	3.	Student shows the number 6 by counting out 6 bears of the same color.
	4.	Student explains the minus sign tells us to subtract.
	5.	Student explains that when we subtract, we take away.
	6.	Student decides to take away 2 of the bears.
	7.	Student counts 2 of the bears to take away.
	8.	Student counts 1, 2 and moves those bears to the side.
	9.	The answer is the number of bears left. The student counts 1, 2, 3, 4.
	10	The answer to $6 - 2 = 4$ .
	11	. When we subtract, the amount gets smaller.

(Teacher should prompt students through the sequence of activities as needed.)

#### **Corrective Feedback:**

#### Example 1

Student demonstrates incorrect counting of concrete manipulatives and struggles with one-to-one correspondence.

Teacher demonstrates hand over hand counting. Teacher and student count together. Make sure student demonstrates he or she can count before moving forward. Teacher redirects and models counting the bears one at a time. Teacher encourages student to slow down while counting.

### Activity Two: Solve Subtraction Problem as an Unknown-Addend Problem

Purpose:	Develop understanding of the relationship between addition and subtraction
Principles of Intensive Intervention	<ul> <li>Provide concrete learning opportunities (including use of manipulatives).</li> </ul>
Illustrated:	<ul> <li>Provide explicit error correction, and have students repeat the correct process.</li> </ul>
	<ul> <li>Use precise, simple language to teach key concepts or procedures.</li> </ul>
	<ul> <li>Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.</li> </ul>
	<ul> <li>Provide repeated opportunities to practice each step correctly.</li> </ul>
Materials:	<ul> <li>One-sided flashcards with subtraction problems (minuends should not exceed 18 and subtrahends should fall between 0–9; see Supplemental Materials)</li> </ul>
	<ul> <li>Concrete objects (teachers choice)</li> </ul>
	<ul> <li>Worksheet: Basic Facts: Addition Concepts (for extra practice)</li> </ul>
	• Worksheet: Basic Facts: Subtraction Concepts (for extra practice)
Modeling:	1. Present subtraction problem $(12 - 7 = )$
	2. Use colored bears or other concrete manipulative.
	3. Show the number 12 by counting out 12 bears of the same color.
	4. Explain to students that the minus sign tells us to subtract.
	5. When we subtract, we take away, and the amount gets smaller.

- 6. To solve the problem, we take away 7 of the bears.
- 7. Let's count 7 of the bears to take away.
- 8. Teacher and students count 1, 2, 3, 4, 5, 6, 7.
- 9. Say, "Our answer is the number of bears we have left. Let's count together."
- 10. Say, "The answer to 12 7 = 5." Write the answer: 5.
- 11. Explain that you can also think of 12 7 as an addition problem. 7 + (? or the answer to the subtraction problem) = 12.
- 12. Demonstrate with the bears.
- 13. Count 12 bears (use red).
- 14. Count 7 bears (use yellow).
- 15. Line up the 7 yellow bears underneath the 12 red bears.
- 16. Place 5 green bears under the remaining red bears.
- 17. Explain to students that there are 12 red bears in this row.
- 18. This row shows 7 yellow bears plus 5 green bears.
- 19. You can solve the problem 12 7 = 5 by thinking about adding.
- 20. 7 + 5 = 12, which is what the green and yellow bears show when they are put together.
- 21. Explain that this is how you can use addition to check your subtraction work.

#### **Guided Practice:** 1. Teacher presents subtraction problem (11 - 8 = )

- 2. Use colored bears or another concrete manipulative.
- 3. Students show the number 11 by counting out 11 bears of the same color.
- 4. Students explain that the minus sign tells us to subtract.
- 5. Students explain subtraction means to take away, and the amount gets smaller.
- 6. Students take away 8 of the bears.
- 7. Students counts 1, 2, 3, 4, 5, 6, 7, 8.
- 8. Students explain the answer to 11 8 = 3. Write the answer: 3.
- Teacher prompts student to now think of 11 8 as an addition problem to solve it.

- 10. Students writes 8 + (? The answer to the subtraction problem) = 11.
- 11. Students lines up 11 bears with 1 color.
- 12. Students show 8 bears underneath the first 8 of the original 11 bears with a different color.
- 13. Students place 3 bears under the remaining original 11 bears.
- 14. Students explain that this shows 11 bears in the first row.
- 15. Students explain the second row shows 8 bears plus 3 bears is the same as 11 bears.
- 16. Student explains you can solve the problem 11 8 = 3 by thinking about adding.
- 17.8 + 3 = 11, which is what the second row shows, so 11 8 = 3.

#### **Corrective Feedback:**

Example 1

Student demonstrates incorrect counting of concrete manipulatives and struggles with one-to-one correspondence.

Teacher redirects and models counting the bears one at a time. Teacher encourages student to slow down while counting. Teacher demonstrates hand over hand counting. Teacher and student count together. Make sure student demonstrates he or she can count before moving forward.

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### Worksheet

### **Basic Facts: Addition Concept**

**Objective:** Add using concrete objects or representative drawings to find the sum.

**Directions:** Using manipulatives or drawings (e.g., circles, tick marks), students should count out the value of each of the addends in order to determine the sum.

#### Addition:



3	+	0	=	
			_	
	Ť		-	

2	+	5	=	
			_	
	+		=	

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4	+	3	=	
	+		=	

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### Worksheet

## **Basic Facts: Subtraction Concepts**

**Objective:** Subtract using concrete objects or representative drawings to find the difference.

**Directions:** Using manipulatives or drawings (e.g., circles, tick marks), students should count out the value of the minuend and then remove the value of the subtrahend in order to find the difference.

Subtraction:



4	-	1	=	
			_	
			-	

9	-	3	=	
			=	

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7	-	5	=	
			=	

## **3. Basic Fact Fluency:** Addition and Subtraction

#### **Sample Activities**

a) Activity One: Addition and Subtraction Flashcards: Know it or Find It

#### Worksheets

- a) Basic Fact Fluency: Addition
- b) Basic Fact Fluency: Subtraction





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## **Sample Basic Fact Fluency:** Addition and Subtraction Activity 1

#### **College- and Career-Ready Standard Addressed:**

1.0A.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making 10 (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a 10 (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows that 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Activity One:	Addition and Subtraction Flashcards: Know it or Find it
Purpose:	Develop fluency with retrieval of basic addition and subtraction facts.
Principles of Intensive Intervention Illustrated:	<ul> <li>Provide concrete learning opportunities (including use of manipulatives).</li> <li>Provide explicit error correction, and have students repeat the</li> </ul>
	correct process.
	Use precise, simple language to teach key concepts or procedures.
	<ul> <li>Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.</li> </ul>
	Provide repeated opportunities to practice each step correctly.
Materials:	<ul> <li>Double-sided flashcards with addition and subtraction problems and answers (totals should not exceed 20; minuends should not exceed 20; see Supplemental Materials)</li> </ul>
	<ul> <li>Scorecard or chart (see Supplemental Materials)</li> </ul>
	<ul> <li>Worksheet: Basic Fact Fluency: Addition (for extra practice)</li> </ul>
	<ul> <li>Worksheet: Basic Fact Fluency: Subtraction (for extra practice)</li> </ul>
	Stopwatch or timer

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Modeling:	1.	Explain that the students will take turns solving addition and subtraction problems using flashcards.
	2.	Teacher explains as a group (this works best in a group of 3 students) that students will try to answer as many flashcards as they can correctly in 2 minutes.
	3.	The goal of the game is for students to improve their scores every time they play.
	4.	First, the teacher explains the fastest way to answer a problem is to <b>know</b> it right away.
	5.	If a student knows it, he or she states the answer, and then it's the next person's turn.
	6.	If a student does NOT know it, he or she must <b>find</b> the answer before moving to the next person.
	7.	<ul> <li>Teacher reviews strategies for finding the answer. These include:</li> <li>Counting on (using fingers; see Step 8)</li> <li>Using scratch paper to find the answer</li> <li>Any other useful strategy the teacher deems reasonable.</li> </ul>
	8.	<ul> <li>Teacher gives an example of all the different ways to find the answer.</li> <li>Counting On Strategy (Example: 7 + 2 = ) <ul> <li>Start with the bigger number (7).</li> <li>Count on the other number (2, so student counts on 2 from 7: 8, 9).</li> <li>Last number counted (9) is the answer!</li> </ul> </li> <li>Counting On Strategy (Example: 7 - 2 = ) <ul> <li>Start with the minus number (2).</li> <li>Count on to the other number with fingers (7, so the student counts from 2 to 7).</li> <li>Number of fingers (5) is the answer!</li> </ul> </li> </ul>
Guided Practice	1.	Teacher explains they'll practice the flashcard activity before doing the activity for the timed 2 minutes.
	2.	Student presents basic fact flashcard to first student in the group.
	3.	If student knows it, the next student gets a flashcard.

- 4. If student does not know it, he or she finds the answer.
  - Teacher provides help with counting on strategy (using fingers)
  - Teacher also demonstrates how to make marks to add with scratch paper
- 5. This process is repeated until all students in the group have completed one flashcard per person.
- 6. Students play the game for 2 minutes.
- 7. Teacher records the number of correct problems and puts the information on a chart.
- 8. Students try to beat their score in subsequent games.
- 9. The score is for the group.

(If only one student needs fluency practice, this would be an individual score.)

#### **Corrective Feedback:**

Corrective feedback is built in to the activity to happen immediately when mistakes are made. (See Modeling, Step 7.) The immediate corrective feedback causes the student to take more time during the activity; thereby, lowering the score.

### **Basic Fact Fluency: Addition**

**Objective:** Develop fluency with retrieval of basic addition and subtraction facts.

		Addition		
10	9	0	4	15
+ 1	+ 6	+ 3	+ 6	+14
1 3	4	2	3	5
+ 4	+ 5	+ 4	+ 7	+ 1 3
10	0	5	3	1
+10	+ 6	+ 5	+ 9	+ 4
9+4	1 2	8	7	3
	+ 6	+ 4	+ 1 2	+ 1 4
8	2	1	5	1
+ 0	+ 1 3	+ 1	+ 1 0	+ 0
7	2	5	0	3
+ 9	+ 1 0	+ 3	+ 7	+ 6
9 + 9	4	10	7	8
	+ 1 1	+ 3	+ 6	+ 1 0
4	7	15	8	1
+ 1 2	+ 2	+ 0	+ 5	+ 7







### **Basic Fact Fluency: Addition**

**Objective:** Develop fluency with retrieval of basic addition and subtraction facts.

Addition					
3	7	11	6	13	
+ 3	+ 8	+ 6	+ 8	+ 6	
2	14	6	4	1	
+ 2	+ 1	+ 7	+ 2	+ 1 9	
4	19	6	18	1	
+ 8	+ 0	+ 4	+ 1	+ 5	
9	4	16	5	15	
+ 3	+ 9	+ 4	+ 8	+2	
9	3	11	6	7	
+ 8	+ 1 0	+ 4	+ 1 2	+ 0	
7 + 3	5	7	9	14	
	+ 2	+ 1 0	+ 1 0	+ 5	
5	4	1	1	6	
+ 9	+ 0	+ 1 3	+ 1 7	+ 1 0	
3	1	9	2	6	
+ 1 6	+ 8	+ 0	+ 3	+ 1	

### **Basic Fact Fluency: Addition**

**Objective:** Develop fluency with retrieval of basic addition and subtraction facts.

		Addition		
3	6	5	6	4
+ 2	+ 6	+ 7	+ 4	+ 7
7	17	9	2	1
+ 7	+ 2	+ 2	+ 1 6	+ 6
8	8	3	6	1
+ 8	+ 9	+ 1 2	+ 2	+ 1 6
4	15	1	12	4
+ 1 0	+ 3	+ 2	+ 6	+ 3
15	9	4	8	2 0
+ 5	+ 1	+ 4	+ 1 1	+ 0
6	1 4	1 1	2	18
+ 3	+ 2	+ 7	+ 1 3	+ 2
3	8 + 7	9	6	8
+ 5		+ 5	+ 0	+ 3
6	3	8	17	1
+ 5	+ 1	+ 2	+ 0	+ 6

Worksheet: Basic Fact Fluency: Addition

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### Worksheet

### **Basic Fact Fluency: Subtraction**

**Objective:** Develop fluency with retrieval of basic addition and subtraction facts.

**Directions:** After practicing with flashcards, attempt to complete as much of a worksheet as possible in two minutes.

		Subtraction		
17	6	1 4	2	11
- 9	- 6	- 1 0	- 0	- 7
10	8	3	7	9
- 0	- 5	- 3	- 1	- 4
12	20	17	1	9
- 1	- 5	- 5		<u>- 3</u>
10	9	16	15	19
- 7	- 1	- 8	- 5	- 8
6	13	8	6	18
- 0	- 8	- 2	<u>- 5</u>	- 0
16	<u> </u>	2	12	8
-10		- 2	- 6	- 3
- <u>2</u>	4	- 7	1	15
	- 1	- 7	- 1	- 3
5	7	3	10	20
- 4	- 6	- 0	- 5	- 2

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### **Basic Fact Fluency: Subtraction**

**Objective:** Develop fluency with retrieval of basic addition and subtraction facts.

	Subtraction					
2	2 0	11	7	9		
- 1	- 7	- 5	- 3	- 2		
11	<u> </u>	19	9	18		
- 1		- 2	<u>- 5</u>	- 7		
4	9	6	14	16		
- 2	- 8	- 4	- 0	- 9		
8	1 2	9	15	13		
- 4	- 3	- 6	-10	- 9		
0	4	6	17	3		
- 0	- 3	- 1	- 2	- 2		
10	1 3	19	10	13		
-10	- 2	- 5	- 4	- 6		
7	1 8	7	4	17		
- 2	- 4	- 5		- 6		
15	10	8	16	2 0		
- 8	- 3	- 7	- 6	- 3		

### **Basic Fact Fluency: Subtraction**

**Objective:** Develop fluency with retrieval of basic addition and subtraction facts.

	Subtraction					
10	8	9	16	7		
- 6	- 8	- 3	- 5	- 0		
14	5	2	3	15		
- 9	- 3	- 0	- 1	- 4		
5	1 0	6	13	9		
- 5	- 8	- 2	- 5	- 7		
<u>    1  2</u>	1 6	19	2 0	11		
0	- 2	- 7	- 8	- 3		
17	4	9	5	1 4		
- 8	- 4	- 5	- 0	- 7		
19	10	8	9	17		
- 3	- 1	- 6	- 9	- 1		
13	1 1	1 8	4	6		
- 1	- 9	- 8	- 2	- 3		
1 2	9	7	5	15		
- 5	- 0	- 4	- 1	- 0		

Worksheet: Basic Fact Fluency: Subtraction
# 4. Basic Facts: Multiplication and Division Concepts

### **Sample Activities**

- a) Activity One: Representing Multiplication Concepts with Concrete Objects
- b) Activity Two: Representing Division Concepts with Concrete Objects

### Worksheets

- a) Basic Facts: Multiplication Concepts
- b) Basic Facts: Division Concepts





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# **Sample Basic Facts:**

## Multiplication and Division Concepts Activity 1–2

### **College- and Career-Ready Standard Addressed:**

3.0A Represent and solve problems involving multiplication and division.

Interpret products of whole numbers (e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each and interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).

Activity One:	Representing Multiplication Concepts
	with Concrete Objects
Purpose:	To multiply using concrete objects

Principles of Intensive Intervention Illustrated:	<ul> <li>Provide concrete learning opportunities (including use of manipulatives).</li> <li>Provide explicit error correction, and have students repeat the</li> </ul>
	correct process.
	Use precise, simple language to teach key concepts or procedures.
	<ul> <li>Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.</li> </ul>
	<ul> <li>Provide repeated opportunities to practice each step correctly.</li> </ul>
Materials:	<ul> <li>Flashcards with multiplication problems (factors should fall between 0-9; see Supplemental Materials)</li> </ul>
	<ul> <li>Worksheet: Basic Facts: Multiplication Concepts (for extra practice)</li> </ul>
	<ul> <li>Concrete objects (teachers choice)</li> </ul>
	<ul> <li>Small paper plates</li> </ul>







Modeling:	Present multiplication problem $(3 \times 5 = )$	
	. Use colored round chips or other concrete manipulative.	
	Explain that 3 times 5 means $5 + 5 + 5$ or 3 groups of 5.	
	To show 3 times 5, we need 3 small paper plates.	
	. Count 3 plates together. Explain each plate is a "group."	
	. We put 5 chips on each of the 3 plates.	
	Count 1, 2, 3, 4, 5, and place 5 chips on a plate.	
	. Repeat for the remaining plates.	
	Explain you've shown 3 groups of 5.	
	0. Point to the chips on the plate and say, "We have 1, 2, 3, times 1, 2, 3" (point to each plate).	4, 5
	1. The answer is the total number of chips.	
	2. Say, "Let's count the chips together."	
	3. Count together "1, 2, 3, 4,15."	
	$4.3 \times 5 = 15.$	
Guided Practice	Teacher presents multiplication problem $(4 \times 4 = )$ .	
	. Use colored round chips or other concrete manipulative.	
	. Teacher asks student what 4 times 4 means.	
	Student explains that 4 times 4 means 4 + 4 + 4 + 4 or 4 groups with 4 in each group. (This will likely take several session of practice for student to understand meaning of multiplication.)	
	. Teacher asks student to show 4 times 4 with the chips and plates.	
	Student chooses 4 small paper plates.	
	Student counts 4 plates.	
	<ul> <li>Student explains that he or she can show 4 times 4 by pu</li> <li>4 chips on each of the 4 plates.</li> </ul>	tting
	. Student counts 1, 2, 3, 4 chips and places them on on the plates.	e of
	0. Student repeats for the remaining plates.	
	1. Student explains that he or she has shown 4 times 4.	

- 12. The student points to the chips on one plate and says, "We have 1, 2, 3, 4 times 1, 2, 3, 4" (point to each plate).
- 13. The answer is the total number of chips.
- 14. Student counts the chips together.
- 15. Count together "1, 2, 3, 4, . . . 16."
- $16.4 \times 4 = 16.$

### **Corrective Feedback:**

Students should interpret multiplication problems as "\_\_ groups of \_\_." It is important to teach students the commutative property of multiplication (i.e., the order of numbers does not matter). It's easier to do 3 groups of 9 ( $3 \times 9$ ) instead of 9 groups of 3 ( $9 \times 3$ ). The commutative property also allows students to build fluency quickly because if he or she knows  $3 \times 9 = 27$ , then he or she also knows that  $9 \times 3 = 27$ .

Students should also understand multiplication as repeated addition. Using the plates and manipulatives helps with understanding 3 times 5 is 1, 2, 3, 4, 5. 6, 7, 8, 9, 10. 11, 12, 13, 14, 15.

### Activity Two: Representing Division Concepts with Concrete Objects

Purpose:	To divide using concrete objects							
Principles of Intensive Intervention	<ul> <li>Provide concrete learning opportunities (including use of manipulatives).</li> </ul>							
mustrateu.	<ul> <li>Provide explicit error correction, and have students repeat the correct process.</li> </ul>							
	<ul> <li>Use precise, simple language to teach key concepts or procedures.</li> </ul>							
	<ul> <li>Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.</li> </ul>							
	<ul> <li>Provide repeated opportunities to practice each step correctly.</li> </ul>							
Materials:	<ul> <li>Flashcards with division problems (Problems presented should all work out evenly; dividends should be divided by a factor of that number; divisors should fall between 1–9; see Supplemental Materials)</li> </ul>							
	<ul> <li>Worksheet: Basic Facts: Division Concepts (for extra practice)</li> </ul>							
	<ul> <li>Concrete objects (teachers choice)</li> </ul>							

Small paper plates

Modeling:	1.	Present division problem $(12 \div 4 = )$
	2.	Use colored round chips or other concrete manipulative.
	3.	Explain that $12 \div 4$ is 12 divided into 4 equal groups.
	4.	To show 12 divided by 4, we need 12 chips and 4 smaller plates.
	5.	Count 12 chips. Count 4 small plates.
	6.	Teacher explains that to divide correctly we take 1 chip at a time and divide them equally among the small plates.
	7.	We go in order until we've placed all chips on plates.
	8.	Put 1 chip on each plate. Explain that now we go back to the first plate and continue distributing 1 chip at a time.
	9.	We repeat this process until all 12 chips are placed on plates.
	10	. Explain you've shown 12 divided into 4 equal groups.
	11	. Point to the chips on the plate and say, "We have have 1, 2, 3 chips on one plate."
	12	. The answer is the number of chips on one plate.
	13	$. 12 \div 4 = 3$
Guided Practice	1.	Teacher presents division problem $(18 \div 6 = )$
Guided Practice	1. 2.	Teacher presents division problem $(18 \div 6 = )$ Use colored round chips or other concrete manipulative.
Guided Practice	1. 2. 3.	Teacher presents division problem $(18 \div 6 = )$ Use colored round chips or other concrete manipulative. Teacher asks student to explain what 18 divided 6 means.
Guided Practice	1. 2. 3. 4.	Teacher presents division problem $(18 \div 6 = )$ Use colored round chips or other concrete manipulative. Teacher asks student to explain what 18 divided 6 means. Student explains that $18 \div 6$ is 18 divided into 6 equal groups.
Guided Practice	1. 2. 3. 4. 5.	Teacher presents division problem $(18 \div 6 = )$ Use colored round chips or other concrete manipulative. Teacher asks student to explain what 18 divided 6 means. Student explains that $18 \div 6$ is 18 divided into 6 equal groups. Teacher asks what student needs to show 18 divided into 6 groups.
Guided Practice	1. 2. 3. 4. 5.	<ul> <li>Teacher presents division problem (18 ÷ 6 = )</li> <li>Use colored round chips or other concrete manipulative.</li> <li>Teacher asks student to explain what 18 divided 6 means.</li> <li>Student explains that 18 ÷ 6 is 18 divided into 6 equal groups.</li> <li>Teacher asks what student needs to show 18 divided into 6 groups.</li> <li>Student explains that to show 18 divided by 6, he or she needs 18 chips and 6 small plates.</li> </ul>
Guided Practice	1. 2. 3. 4. 5. 6.	<ul> <li>Teacher presents division problem (18 ÷ 6 = )</li> <li>Use colored round chips or other concrete manipulative.</li> <li>Teacher asks student to explain what 18 divided 6 means.</li> <li>Student explains that 18 ÷ 6 is 18 divided into 6 equal groups.</li> <li>Teacher asks what student needs to show 18 divided into 6 groups.</li> <li>Student explains that to show 18 divided by 6, he or she needs 18 chips and 6 small plates.</li> <li>Student counts 18 chips and 6 smaller plates.</li> </ul>
Guided Practice	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> <li>5.</li> <li>6.</li> <li>7.</li> <li>8.</li> </ol>	<ul> <li>Teacher presents division problem (18 ÷ 6 = )</li> <li>Use colored round chips or other concrete manipulative.</li> <li>Teacher asks student to explain what 18 divided 6 means.</li> <li>Student explains that 18 ÷ 6 is 18 divided into 6 equal groups.</li> <li>Teacher asks what student needs to show 18 divided into 6 groups.</li> <li>Student explains that to show 18 divided by 6, he or she needs 18 chips and 6 small plates.</li> <li>Student counts 18 chips and 6 smaller plates.</li> <li>Teacher asks what to do next.</li> </ul>
Guided Practice	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> <li>5.</li> <li>6.</li> <li>7.</li> <li>8.</li> <li>9.</li> </ol>	<ul> <li>Teacher presents division problem (18 ÷ 6 = )</li> <li>Use colored round chips or other concrete manipulative.</li> <li>Teacher asks student to explain what 18 divided 6 means.</li> <li>Student explains that 18 ÷ 6 is 18 divided into 6 equal groups.</li> <li>Teacher asks what student needs to show 18 divided into 6 groups.</li> <li>Student explains that to show 18 divided by 6, he or she needs 18 chips and 6 small plates.</li> <li>Student counts 18 chips and 6 smaller plates.</li> <li>Teacher asks what to do next.</li> <li>Student explains that he or she will place one chip at a time on the smaller plates and repeat until he or she has used all 18 chips.</li> </ul>
Guided Practice	<ol> <li>1.</li> <li>2.</li> <li>3.</li> <li>4.</li> <li>5.</li> <li>6.</li> <li>7.</li> <li>8.</li> <li>9.</li> <li>10</li> </ol>	<ul> <li>Teacher presents division problem (18 ÷ 6 = )</li> <li>Use colored round chips or other concrete manipulative.</li> <li>Teacher asks student to explain what 18 divided 6 means.</li> <li>Student explains that 18 ÷ 6 is 18 divided into 6 equal groups.</li> <li>Teacher asks what student needs to show 18 divided into 6 groups.</li> <li>Student explains that to show 18 divided by 6, he or she needs 18 chips and 6 small plates.</li> <li>Student counts 18 chips and 6 smaller plates.</li> <li>Teacher asks what to do next.</li> <li>Student explains that he or she will place one chip at a time on the smaller plates and repeat until he or she has used all 18 chips.</li> <li>Student puts 1 chip on each plate and repeats this process until all 18 chips are placed on one of the small plates.</li> </ul>

- 12. Student explains he or she has shown 18 divided into 6 equal groups.
- 13. Teacher asks student to explain  $18 \div 6$ .
- 14. Student points to the plate and says, "We have 1, 2, 3 chips on one plate."
- 15. The answer is the number of chips on one plate.
- 16.  $18 \div 6 = 3$

### **Corrective Feedback:**

Students may struggle with distributing the chips *equally* among the groups. Emphasize placing one chip on each plate and then going back and doing that again and again until all the chips have been distributed.

Students may also struggle with the answer. Emphasize the answer is the number of chips on one plate.

### Adaptation

The activities described above model multiplication and division concepts through concrete objects. Once students are comfortable with these activities, teachers should move students toward representational and abstract understanding of multiplication and division. This is the concrete-representational-abstract (CRA) sequence. See below for steps to modify both activities for a representational lesson.

- 1. Instead of plates and chips, teachers should demonstrate how to make circular regions on scratch paper for the plates, and how to use hash marks, X's, or dots for the chips.
- 2. Teacher should demonstrate the same procedures for counting and making sure the marks are distributed equally within the regions.

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# Worksheet

# **Basic Facts: Multiplication Concepts**

**Objective:** Multiply by placing concrete objects into groups on small plates to find the product.

**Directions:** Have student set out number of plates equal to the first factor and then fill each plate with the number of small concrete objects (counting bears, etc.) equal to the second factor. Finally, have the student count the total number of objects in order to determine the product of the equation.

*Optional:* Have student draw in the space provided instead of objects to represent the problem.







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3	x	6	=	

X	7	=	
	X	x 7	x 7 =

4	x	0	=	

5	x	6	=	

9	x	4	=	

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# Worksheet

# **Basic Facts: Division Concepts**

**Objective:** Divide by placing concrete objects into groups on small plates to find the quotient.

**Directions:** Have student count out a number of concrete objects equal to the dividend and then count out a number of plates equal to the divisor. Student should then place the objects onto the plates, alternating between each plate until all of the objects are gone. However many objects are on one plate is the quotient.

*Optional:* Have student draw in the space provided instead of objects to represent the problem.











8	÷	4	=	

÷	7	=	
	÷	÷ 7	÷ 7 =

32	÷	8	=	

35	÷	5	=	

42	÷	6	=	

21	÷	7	=	

32	÷	8	=	

# **5. Supplemental Materials**

- a) One-sided Addition Flashcards
- b) One-sided Subtraction Flashcards
- c) Double-sided Addition Flashcards with Answers on Back
- d) Double-sided Subtraction Flashcards with Answers on Back
- e) Score Cards
- f) One-sided Multiplication Flashcards
- g) One-sided Division Flashcards



































# 0+10=
















## **10 - 9 = 10 - 10 =**



























Supplemental Materials: Double-sided Addition Flashcards with Answers on Back

















Supplemental Materials: Double-sided Addition Flashcards with Answers on Back
































Supplemental Materials: Double-sided Subtraction Flashcards with Answers on Back















## 10 – 9 = 10 – 10 =

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Student Name		
Round 1		
Round 2		
Round 3		
Round 4		
Round 5		
TOTAL		

Student Name		
Round 1		
Round 2		
Round 3		
Round 4		
Round 5		
TOTAL		

Student Name		
Round 1		
Round 2		
Round 3		
Round 4		
Round 5		
TOTAL		



Supplemental Materials: One-sided Multiplication Flashcards



































