Error Analysis in Math

In the data-based individualization framework, if a student demonstrates limited response to a validated intervention delivered with fidelity, educators use diagnostic data to determine how to best intensify the intervention. Errors in student work samples are one type of diagnostic data that educators may use to better understand students' skill gaps. **Error analysis** is a process for analyzing student work samples to (a) identify common types of errors in the student's work and (b) use this information to better align the intervention with the student's needs. This tip sheet provides an overview of how to analyze errors in math.

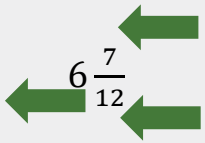
1. Select student work to analyze.

Progress monitoring assessments, curriculum-based assessments, worksheets, or intervention practice problems are all excellent resources for error analysis. For the best results, select student work that includes at least three to five math problems of the same type (e.g., multidigit addition, addition of fractions with common denominators; Brown & Skow, 2016).

2. Evaluate each numeral in the student's answer.

Mark each incorrect numeral in the student's answer (as opposed to marking the whole answer incorrect) to help you quickly understand the error patterns. Score addition, subtraction, and multiplication problems from right to left and division problems from left to right. If the student has used a partial algorithm (partial sums, partial products) rather than a traditional or standard algorithm, then addition, subtraction, multiplication, and division problems should be scored left to right (Brown & Skow, 2016). Exhibit 1 shows examples of how to identify the number of correct digits in math computation problems.

Exhibit 1. Correct and Incorrect Digits in Math Computation Problems

Problem type	How to score each digit	Example
Addition, subtraction, multiplication	Score each correct digit in the answer from *right to left*. 417 	Correct answer: 417. Student answers: 417 (3 correct digits) 415 (2 correct digits) 47 (1 correct digit; the 4 is not in the correct position when scoring left to right)
Division	Score each correct digit in the answer from *left to right*, with remainders scored from *right to left.* 36 R 13 	Correct answer: 36 R 13. Student answers: 37 R 1 (1 correct digit; the 1 in the remainder is not in the correct position when scoring right to left) 26 R 23 (2 correct digits)
Decimals	Start at the decimal point and work outward in both directions. 83.76 	Correct answer: 83.76 Student answers: 8.6 (0 correct digits; the 8 is not in the correct position when scoring from right to left) 84.7 (2 correct digits)
Fractions	Score correct digits in each part of the problem (whole number, numerator, and denominator) from *right to left,* then add them for total correct digits. 	Correct answer: $6\frac{7}{12}$ Student answers: $6\frac{8}{11}$ (2 correct digits) $5\frac{6}{12}$ (2 correct digits)

3. Identify error patterns and plan targeted instruction.

After evaluating each numeral in the student's responses, further analyze the student work to learn more about the types of errors the student made. Exhibit 2 shows examples of analyzing errors to learn more about students' mathematical thinking and application of math skills.

Exhibit 2. Error Analysis Examples

Correct answer	Student A answer	Student B answer	Teacher analysis
3058 + 1148 4206 (4 correct digits)	3058 + 1148 ~~4196~~ (2 correct digits)	3058 + 1148 ~~4207~~ (3 correct digits)	▪ Student A appears to have not regrouped the ten ones when adding 8 + 8 = 16, which resulted in incorrect digits in the tens and hundreds places. ▪ Student B regrouped the ten ones when adding 8 + 8 and added correctly in the tens, hundreds, and thousands columns. However, the student incorrectly added 8 + 8 in the ones column.
$1\frac{2}{3} + 3\frac{2}{3} = 5\frac{1}{3}$ (3 correct digits)	$1\frac{2}{3} + 3\frac{2}{3} = \frac{4}{6}$ (0 correct digits)	$1\frac{2}{3} + 3\frac{2}{3} = 4\frac{4}{3}$ (1 correct digit)	▪ Student A added the digits in each place—whole number, numerator, and denominator—instead of only adding the numerators of the fractions. ▪ Student B added correctly but did not simplify the answer.
3.9 + 4.3 8.2 (2 correct digits)	3.9 + 4.3 ~~7.12~~ (0 correct digits)	3.9 + 4.3 ~~7~~.2 (1 correct digit)	▪ Neither student regrouped the tenths to the ones place correctly. ▪ Student A seems to know that 9 + 3 = 12, but placing 12 after the decimal point suggests the student does not understand decimal-place values and the need to regroup. ▪ Student B put the 2 digit in the tenths place correctly but did not regroup the tenths to the ones place.

When analyzing student work, it is helpful to classify errors by type so that you can target instruction on these error types. There are multiple ways to classify the types of errors. One approach is to classify the errors as slips, bugs, or neither. The information below is based on the video Error Analysis: What Are Slips and Bugs? (Math SPIRAL, 2024).

Error Type 1: Slip

A **slip** is a careless error that is often caused by poor organization. The student can perform the skill correctly but made a careless error. As seen in Exhibit 3, the teacher may identify a slip if a student consistently shows that they know the skill across multiple problems but make an error on one problem. In this case, the teacher may hypothesize that they need to explicitly teach the student to organize their work, write neatly, and check for mistakes. Additionally, the teacher may instruct the student to use a self-monitoring checklist to help minimize slips.

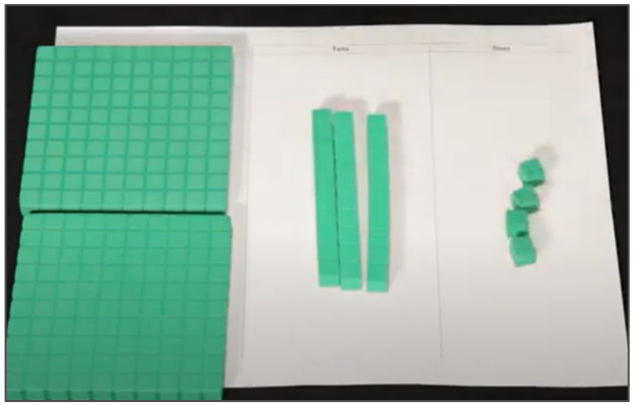
Exhibit 3. Strategies for Addressing Math Slips

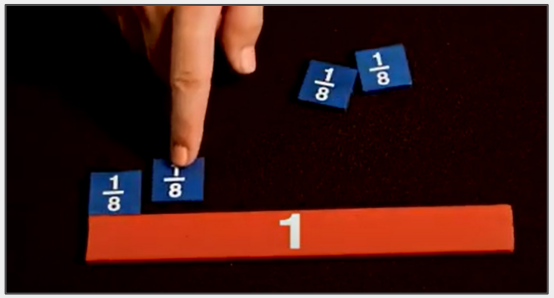
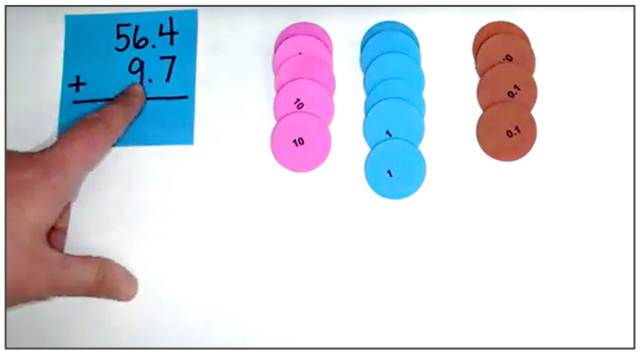
Example from student work	Teacher analysis	Strategy	Example of strategy application
1 18 + 72 90 1 34 + 69 103 11 606 + 98 704 50 + 12 2 62	The student understands regrouping when adding (as shown by the first three problems) but made a careless error by not lining up the digits in ones and tens place value columns correctly in the fourth problem.	▪ Line up numbers by place value and draw a line to separate the columns, or use graph paper to line up place value columns. ▪ Use self-monitoring checklists.	50 12 + 2 Turn in this checklist with your assignment. Name_____ Date_____ 1. My digits are written in place-value columns. Y ? N 2. Others can read my numerals. Y ? N 3. Sometimes I was stuck. Y ? N 4. I checked my answers. Y ? N 5. Describe a situation in which this computation could be used. Y ? N _____ _____

Error Type 2: Bug

A **bug** is a misunderstanding of a math concept. With errors that are bugs, you should determine the math misconception that caused the error. If you identify three or more errors due to the same misconception, this is a bug. Exhibit 4 provides three examples of student work that show a bug. In this case, the teacher may hypothesize that they need to focus on providing explicit instruction to address the misconception, using multiple representations and brief, daily fluency practice (once the student is proficient with the skill).

Exhibit 4. Strategies for Addressing Math Bugs

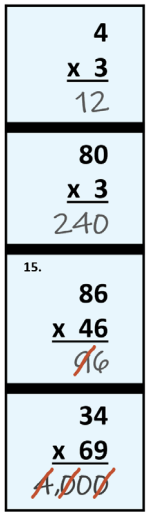
Example from student work	Teacher analysis	Strategy	Example of strategy application
800 − 692 = 292 70 − 19 = 69 206 − 35 = 231	The student has a misconception that digits from the minuend (i.e., the number from which you are subtracting) are interchangeable with digits from the subtrahend (i.e., the number you are subtracting from the minuend).	Provide additional explicit instruction with guided practice on the concept of subtraction with regrouping.	Use base ten blocks to model subtraction with regrouping. Click here for a video example. 

Example from student work	Teacher analysis	Strategy	Example of strategy application
$1\frac{2}{3} + 3\frac{2}{3} = 4\frac{4}{6}$ $2\frac{1}{4} + 3\frac{2}{4} = 5\frac{3}{8}$ $3\frac{3}{8} + 1\frac{2}{8} = 4\frac{5}{16}$	The student has a misconception that you can add digits in each place—whole number, numerator, and denominator—instead of only adding the numerators of the fractions.	Provide additional explicit instruction with guided practice on the concept of adding fractions with common denominators.	Use fraction tiles to model adding fractions with common denominators. Click here for a video example. 
3.9 + 4.3 = 7.12 21.5 + 2.8 = 23.13 7.9 + 5.6 = 12.15	The student does not understand decimal place value and the need to regroup the values in the tenths place.	Provide additional explicit instruction with guided practice on the concept of regrouping decimals when adding.	Use place value disks to model addition of decimals with regrouping. Click here for a video example. 

Error Type 3: Neither Slip nor Bug

With some errors, there is not an identifiable slip or bug that is causing the error. Instead, the student does not have the prerequisite skills to solve the problem. For example, before a student can add two-digit numbers, they must first know math facts. In this case, the teacher may hypothesize that they need to identify which skills the student does and does not know and begin instruction by addressing the gaps in the student's prerequisite skills. Exhibit 5 provides an example of a student who may need additional support in a prerequisite skill.

Exhibit 5. Using a Math Intervention Skill Tracker to Plan Instruction

<table>
<tr><th>Example from student work</th><th>Teacher analysis</th><th>Strategy</th><th>Example</th></tr>
<tr>
<td> 
</td>
<td>The student can correctly solve one-digit by one-digit and one-digit by two-digit multiplication problems but does not correctly solve two-digit by two-digit multiplication problems.</td>
<td>Use a math intervention skill tracker to identify the multiplication skills in which the student is proficient.
Click here for Math SPIRAL intervention trackers, under the Resources tab.</td>
<td>The student needs explicit instruction in multiplying two-digit numbers by two-digit numbers.
<table>
<tr><th colspan="4">Intervention Guide</th></tr>
<tr><th></th><th>Description</th><th>Example</th><th>☑</th></tr>
<tr><td>A</td><td>Multiplying one-digit numbers by one-digit numbers</td><td>4 × 8</td><td>✓</td></tr>
<tr><td>B</td><td>Dividing one-digit numbers by one-digit numbers</td><td>9 ÷ 3</td><td></td></tr>
<tr><td>C</td><td>Multiplying one-digit numbers by two-digit numbers</td><td>4 × 76</td><td>✓</td></tr>
<tr><td>D</td><td>Dividing two-digit numbers by two-digit numbers</td><td>98 ÷ 7</td><td></td></tr>
<tr><td>E</td><td>Multiplying two-digit numbers by two-digit numbers</td><td>56 × 19</td><td></td></tr>
<tr><td>F</td><td>Dividing three-digit numbers by one-digit numbers</td><td>675 ÷ 5</td><td></td></tr>
<tr><td>G</td><td>Multiplying one-digit numbers by three-digit numbers</td><td>8 × 349</td><td></td></tr>
<tr><td>H</td><td>Dividing four-digit numbers by one-digit numbers</td><td>1,898 ÷ 2</td><td></td></tr>
<tr><td>I</td><td>Multiplying one-digit numbers by four-digit numbers</td><td>3 × 2,410</td><td></td></tr>
<tr><td>J</td><td>Dividing four-digit numbers by two-digit numbers – 5th Grade Standard</td><td>7,240 ÷ 40</td><td></td></tr>
<tr><td>K</td><td>Multiplying two-digit numbers by three-digit numbers – 5th Grade Standard</td><td>78 × 902</td><td></td></tr>
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4. If needed, conduct informal diagnostic observations and interviews.

If you are unsure about the student's understanding of the math concept or the cause of math errors, a brief informal observation and interview can be a helpful way to gather more information. You can ask the student questions to gain insight into their approach to solving the problem. Here are some tips for conducting interviews:

- Have a problem(s) ready for the student to complete during the interview.
- Use the interview to gather information, not as an opportunity for instruction.
- Accept student responses without judgment, and keep your responses neutral.
- Ask probing questions about both incorrect and correct answers.

To interview the student, use a simple script with some of the suggested questions in Exhibit 6.

Exhibit 6. Sample Script and Interview Questions

Example script	Example interview questions and prompts
▪ Read the problem, please. – What is the question asking you to do? – How are you going to approach the problem? – Do the work and tell me about your thinking as you work. ▪ Write down your answer.	▪ How did you get your answer? I may have missed something. ▪ If you had to teach someone else to solve the problem, what would you tell them? ▪ This time, I'll hold the pencil. You act as the teacher and tell me what to do.

Conclusion

When a student is not making adequate progress in a math intervention, the teacher may hypothesize that the intervention needs to be better aligned with the student's needs. Error analysis can help the teacher identify patterns of errors in the student's work to better align instruction to the student's needs related to math organizational skills, conceptual misunderstandings, and missing prerequisite skills.

References

Brown, J., & Skow, K. (2016). *Mathematics: Identifying and addressing student errors.* IRIS Center https://iris.peabody.vanderbilt.edu/wp-content/uploads/pdf_case_studies/ics_matherr.pdf

Math SPIRAL. (2024, October 24). *Error analysis: What are slips and bugs?* [Video]. YouTube. https://www.youtube.com/watch?v=DFqDphVZLmg

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